Bloomberg’s Portfolio and Risk Analytics solution, available via the Bloomberg Professional® service, offers a comprehensive set of customizable tools for the desktop. In today’s market, your company’s performance depends on the ability to understand and manage risk and consistently out-think the market. Bloomberg supports portfolio managers, risk managers and senior management with a series of new and enhanced tools to systematically analyze and track portfolio risk, and to construct and rebalance portfolios that optimally achieve investment objectives and criteria.

PORTFOLIO VALUE-AT-RISK
The new VAR tab is a part of Bloomberg’s suite of portfolio and risk analytics provided by PORT <GO>. It enables risk managers and portfolio managers to analyze the tail risk of their portfolios using the latest risk modeling techniques.

This document describes the new methodology for portfolio value-at-risk (VaR) computation provided by Bloomberg Portfolio and Risk Analytics. Three types of VaR are provided:

1. Parametric VaR
2. Historical VaR
3. Monte Carlo VaR

The new VaR methodology utilizes the factor structure provided by the Bloomberg factor models, in a way that makes the VaR consistent with portfolio tracking error and volatility that are computed using the same factor models. For Historical and Monte Carlo VaR an array of valuation choices are offered, ranging from linear pricing using the Bloomberg factor models to Stress Matrix Pricing (SMP) and full valuation. In the remainder of this document we describe the different components of Bloomberg VaR calculation, namely, the Bloomberg factor models, security valuation methods, and details of Parametric, Historical and Monte Carlo VaR calculation.

BLOOMBERG FUNDAMENTAL FACTOR MODELS
Reliable estimation of portfolio volatility is a key first step towards computing reliable VaR estimates. Bloomberg uses linear factor models to estimate portfolio volatility.

Factor models have become an indispensable tool for modern portfolio management as well as risk management. They provide greater understanding of sources of portfolio risk, and the ability to attribute portfolio performance, to forecast both absolute risk and benchmark-relative risk and to improve portfolio construction. Recent market volatility highlights the importance of controlling unwanted factor exposures in portfolios. While factor models have been in use for at least two decades, the quantitative equity hedge fund meltdown of August 2007, the market collapse in the wake of the Lehman Brothers bankruptcy, and extreme volatility of several factors since then have attracted the attention of traditional and quantitative portfolio managers alike and have dramatically increased client interest in factor models.

Factor models are based on the basic principle that security returns are driven by a set of common factors. Therefore, portfolio risk depends on volatility and correlation of these factors and on the amount of portfolio exposure to individual factors. Additionally, there are risks not captured by the common factors; factor models help estimate these “non-factor” risks as well.

Bloomberg’s approach to constructing risk factor models uses a combination of explicit and implicit factors. An implicit or fundamental factor model is constructed by defining security exposures to each factor and then imputing factor returns from a regression of security returns on the exposures. While this class of models has several advantages over the alternatives, we chose this approach primarily due to its better interpretability by the user. It gives greater insight into the portfolio risk sources and leads to intuitive action items. Additionally, explicit factors are used when the impact of certain observable factors on security returns is known. For example, FX rates are used as explicit factors in equity factor models, and changes in the curve are used in fixed income factor models. Factor exposures for explicit factors are analytically computed. The single-period return of the \( n \)th security in the \( t \)th time period is modeled by Bloomberg factor models as

\[
\tau_{nt} = \sum_{k=1}^{K} X_{nkt} f_{kt} + \epsilon_{nt},
\]

where \( X_{nkt} \) is the exposure of the \( n \)th security to the \( k \)th factor at time \( t \), \( f_{kt} \) is the \( k \)th factor return at time \( t \), \( K \) is the number of factors, and \( \epsilon_{nt} \) is the non-factor return of the \( n \)th security at time \( t \).
The choice of valuation method for a given security aims to
valuation methods described in the following is selected for each
portfolio to construct the entire return distribution. One of the four
security in the portfolio and aggregation of returns across the
An integral part of VaR calculation is the valuation of each
VALUATION METHODOLOGY
following sections.
modeling as part of the VaR methodology, as explained in the
The volatility of a portfolio with weights specified by the vector
returns may be written in matrix notation as
\[ R_t = X_t F_t + \epsilon_t, \]
where \( R_t \) is the vector of \( N \) security excess returns at time \( t \),
\( X_t \) is the \( N \times K \) matrix of factor exposures, \( F_t \) is the vector
of \( K \) factor returns at time \( t \) \( \epsilon_t \) is the vector of \( N \) non-factor
returns at time \( t \).
From this model we can derive the asset return covariance
matrix as
\[ Q_t = X_t \Sigma_t X_t^T + D_t, \]
where \( t \) is the factor return covariance matrix, and \( D_t \) is the
sparse matrix of non-specific returns.
The implicit factor returns at time \( t \) are estimated using weighted
cross-sectional regression of the asset returns on factor
exposures at time \( t \). The estimated factor returns are then used
to estimate the factor covariance matrix \( \Sigma_t \), using exponential
averaging and shrinkage. The choice of half-life for exponential
averaging for factor variances is based on the specific factor
model—please see documentation on the individual Bloomberg
factor models. For example, Bloomberg uses a half-life of 26
weeks for exponential averaging of variances and 52 weeks for
correlations for equity models.
The volatility of a portfolio with weights specified by the vector \( w \)
can be computed using the above factor model as
\[ \sigma_{wt} = \sqrt{w^T Q_t w} = \sqrt{w^T X_t \Sigma_t X_t^T w + w^T D_t w}. \]
Bloomberg factor models are used both for valuation and risk
modeling as part of the VaR methodology, as explained in the
following sections.
VALUATION METHODOLOGY
An integral part of VaR calculation is the valuation of each
security in the portfolio and aggregation of returns across the
portfolio to construct the entire return distribution. One of the four
valuation methods described in the following is selected for each
security based on the type of the security and user preferences.
The choice of valuation method for a given security aims to
achieve computational efficiency without sacrificing accuracy.
Linear Factor Model Pricing
The Bloomberg factor models described previously are used
by this method to compute the return of a security, given factor
returns and the non-factor return. This method is implicitly used
for all securities in computing Parametric VaR, as explained
in Section 4. This method is also used for Historical and Monte
Carlo VaR for securities whose price is accurately modeled as
a linear combination of (explicit or implicit) model factors.
Examples of such securities are equities and fixed income
securities without strong convexity or optionality.
Each scenario in the computation of Historical and Monte
Carlo VaR specifies a set of factor returns and non-factor
returns, which are fed into the factor model to compute the
corresponding security returns.
For securities with optionality, the Delta/Gamma or Duration/
Convexity approximation is used, which approximates the
pricing function by the first-and-second-order terms in its Taylor
expansion. Note that the presence of the second order (gamma
or convexity) term makes this a non-linear approximation. But
this non-linearity is converted to a linear operation on factors by
defining market-wide factors that approximate the per-security
non-linear terms. Consider the example of a vanilla equity option:
the gamma term includes the squared return of the underlying
asset, which we approximate by the “market-square” factor (see
documentation of the Bloomberg factor models). The analogous
factor in the case of fixed income securities is the convexity
factor. This approximation linearizes the pricing function with
respect to the factor returns and makes it suitable for Parametric
VaR computation.
Delta/Gamma Pricing
Delta/Gamma or Duration/Convexity pricing is an available
pricing method for securities with optionality in Historical and
Monte Carlo VaR computation. This is similar to the Delta/
Gamma approximation described for linear factor model pricing
with one exception: When computing VaR using historical to
Monte Carlo simulations we have access to all the underlying
factors for each scenario, which enables us to model the true
non-linearity of the gamma term instead of using market-wide
approximations. For example, in the case of equity options this
non-linearity is converted to a linear operation on factors by
defining market-wide factors that approximate the per-security
non-linear terms. Consider the example of a vanilla equity option:
the gamma term includes the squared return of the underlying
asset, which we approximate by the “market-square” factor (see
documentation of the Bloomberg factor models). The analogous
factor in the case of fixed income securities is the convexity
factor. This approximation linearizes the pricing function with
respect to the factor returns and makes it suitable for Parametric
VaR computation.
Stress Matrix Pricing
The Stress Matrix Pricing (SMP) approach offers a compromise
between the accuracy we achieve using Full Valuation and the
speed of Delta/Gamma pricing. It is available in Historical and
Monte Carlo VaR computation for derivative securities, which
include equity options and interest rate and credit products with
embedded options.
The computational effort required to fully value every scenario, we first store the difference between the true price computed using full valuation and the Delta/Gamma price for a much smaller set of scenarios (the “stress matrix”). We then compute the Delta/Gamma price for each scenario in a simulation set and apply a non linearity correction that is interpolated from the stress matrix. This approach gives us the exact price if the scenario exactly matches one of the stored scenarios, but may result in an interpolation error for other scenarios.

For a detailed explanation of the SMP methodology, please see Bloomberg’s Stress Matrix pricing white paper.

**Full Valuation**

For some securities with highly non-linear pricing functions, such as certain exotic derivatives and short time-to-maturity options, we find that the Delta/Gamma Pricing and SMP approaches do not accurately capture the true distribution of security returns, given a distribution of the underlying risk factors. In such cases we offer the flexibility to use full pricing of the security in all scenarios, when Historical or Monte Carlo VaR is selected.

While this method provides the most accurate return distribution given a distribution of factors, it is also the most computationally expensive, and will be used only when the cost in accuracy is too great for the other methods.

**VAR COMPUTATION**

Bloomberg offers the following three choices for VaR computation. Table 1 on the next page summarizes the parameters and techniques used in computing the different VaR estimates.

**Parametric VaR**

The Parametric VaR methodology follows the traditional approach of assuming a jointly normal distribution among all assets in the portfolio to compute a VaR estimate analytically. It makes use of the Bloomberg factor models that provide the asset covariance matrix in terms of the factor covariance matrix, factor exposures and non-factor variances, and models the factor returns and non-factor returns as jointly normal random variables. These assumptions, coupled with the use of linear pricing (see Section 3), imply a normal distribution for the portfolio return. The standard deviation of the normally distributed portfolio return is the portfolio volatility, which is computed as shown in Section 2.

The advantages of the parametric approach to computing VaR are very high speed of computation and compatibility with traditional reporting systems that include this estimate.

However, as is increasingly recognized by risk practitioners, realized distributions of portfolio returns are significantly non-normal—they exhibit fat-tailed behavior, which means that extreme moves in portfolio return occur with a much larger probability than that predicted by a normal distribution. Therefore, Parametric VaR tends to underestimate VaR at very high confidence levels. Parametric VaR also imposes the restriction of linear pricing, which is not suitable for highly non-linear securities. Historical and Monte Carlo VaR estimates described below aim to overcome these drawbacks of Parametric VaR at the expense of a higher computational cost.

**Historical VaR**

This VaR methodology models fat-tailed behavior of returns by using the distribution of realized (historical) factor returns instead of making the assumption that factor returns are normally distributed. The joint distribution of factor returns is represented by a panel of historical daily returns over multiple years of recorded factor history, which we call historical simulations. We simulate the corresponding non-factor returns by drawing from (fat-tailed) Student’s t distributions whose standard deviations are the current estimates of security non-factor volatilities.

We use the factor returns and non-factor returns for each historical scenario and the current factor exposures to compute the corresponding returns of all securities using one of the pricing methods described in Section 3, and aggregate the returns across the portfolio. This yields a historical sequence of portfolio returns given the current portfolio holdings. Historical VaR is then computed as the desired percentile of the portfolio return distribution, e.g., the 5th percentile portfolio return represents the Historical VaR at the 95% confidence level. Since the return distribution is given by historical returns, the choice of the length of the historical period is a critical input for Historical VaR.

In contrast to Parametric VaR, Historical VaR captures the fat-tailed behavior of portfolio returns and offers the flexibility of using multiple valuation techniques described in Section 3. The main advantage of Historical VaR over Monte Carlo VaR is the fact that it makes no assumptions on the joint return distribution other than that the future return distribution is the same as the historical distribution. This often makes Historical VaR easier to interpret and explain. On the other hand, one may question the validity of using the historical distribution for the distribution of future returns, since current market conditions may be quite different from those experienced in the past. Historical VaR is also limited by the length of historical data chosen for historical simulations; a small number of historical scenarios would lead to a lower statistical confidence in the VaR estimate.
Monte Carlo VaR

The Monte Carlo approach to VaR estimation is to estimate the joint distribution of future factor and non-factor returns and to draw a large number of random simulations from this joint distribution to create Monte Carlo scenarios. This enables us to use a forward-looking distribution of the market rather than a backward-looking distribution that historical simulations represent. It also increases the statistical accuracy of VaR estimation compared with Historical VaR, due to the use of a very large number of scenarios.

In order to provide us with added flexibility when formulating the multivariate distribution required in generating VaR, we separate the modeling of the marginal distribution of each risk factor from that of the dependence structure across factors. Bloomberg uses fat-tailed marginal distributions to model the distribution of each individual factor return, and a fat-tailed copula to model the inter dependence of factors, thus going beyond the assumption of jointly normal factor returns. The marginal distributions of individual factors are modeled from the family of Student’s $t$ distributions. The degrees of freedom parameter of the $t$ distribution, which determines the thickness of its tails, is calibrated to historical factor return data for each factor. Bloomberg uses a normal or a Student’s $t$ copula to model the dependence structure between factors.

One of the advantages of using copulas is that they isolate the dependence structure from the structure of the marginal distributions. This separation allows us flexibility in independently choosing the most appropriate models for the marginal distributions of individual factors and the copula for their interdependence. Bloomberg’s implementation of Monte Carlo VaR follows the steps shown above.

1. Estimate the marginal distributions of individual factors:
   - To capture the fat-tailed behavior of the market we use the Student’s $t$ distribution with appropriate degrees of freedom as a parametric model for the marginal distribution of each risk factor. The family of $t$ distributions generalizes the normal distribution, and includes the normal distribution as a special case. We fit a separate $t$ distribution to each factor in the risk model. The degrees of freedom of the Student’s $t$-distribution, which determines the fatness of its tails, is estimated using the maximum-likelihood method, and the variance is estimated using exponentially weighted moving averaging (EWMA) on historical factor data. See documentation of individual Bloomberg factor models for details on the estimation of factor covariance matrix.

### Table 1: Summary of VaR methodologies

<table>
<thead>
<tr>
<th></th>
<th>Parametric</th>
<th>Historical</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of Simulations</strong></td>
<td>N/A</td>
<td>Historical 1 year: 250</td>
<td>10,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Historical 2 year: 500</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Historical 3 year: 750</td>
<td></td>
</tr>
<tr>
<td><strong>Distributional Assumption</strong></td>
<td>Normal</td>
<td>Empirical</td>
<td>Student’s $t$ Marginal</td>
</tr>
<tr>
<td><strong>Time Series Weights</strong></td>
<td>Exponentially Weighted</td>
<td>Equally Weighted</td>
<td>Exponentially Weighted</td>
</tr>
<tr>
<td><strong>Half-Life</strong></td>
<td>26 Weeks for Volatility and 52 weeks for Correlations</td>
<td>None</td>
<td>26 Weeks for Volatility and 52 Weeks for Correlations</td>
</tr>
<tr>
<td><strong>Time Horizon Projection</strong></td>
<td>Daily VaR is Scaled by the Square Root of the Horizon</td>
<td>Daily VaR is Scaled by the Square Root of the Horizon</td>
<td>Daily VaR is Scaled by the Square Root of the Horizon</td>
</tr>
<tr>
<td><strong>Simulated Non-factor Risk</strong></td>
<td>N/A</td>
<td>Non-factor Volatility From the Multi-factor Risk Models Are Used as the Basis to Simulate Non-factor Returns (Student’s $t$ Assumption for Non-factor Risk)</td>
<td>Non-factor Volatility From the Multi-factor Risk Models Are Used as the Basis to Simulate Non-factor Returns (Student’s $t$ Assumption of Non-factor Risk)</td>
</tr>
</tbody>
</table>
2. Estimate the factor copula: We model the inter dependence structure of the joint distribution of factors using a parametric copula distribution. Bloomberg currently uses a t copula with six degrees of freedom, and will provide a choice to the user in the future to select from a normal or a t copula. The copula is parameterized by its correlation matrix, which we estimate using the historical risk factor data. We use exponential weighting with a half-life of 52 weeks and shrinkage to estimate the copula correlation matrix. The copula correlation matrix provides the basis for the scenario generation of joint returns of multiple risk factors.

3. Generate Monte Carlo simulations: Bloomberg draws 10,000 random simulations of factor returns and non-factor returns from the joint distribution estimated as described above. This is done in two steps: We first sample from the copula distribution to obtain a set of 10,000 scenarios that determine the inter dependence structure of factor and non-factor returns. We then transform these scenarios to a panel of joint factor and non-factor returns using the marginal distributions estimated in Step 1.

Once the panel of Monte Carlo scenarios is generated in this manner, the computation of Monte Carlo VaR follows the same method as Historical VaR: each security is priced using the most appropriate valuation method from Section C, security returns are aggregated to form scenarios of portfolio return using the current portfolio holdings, and Monte Carlo VaR is computed as the desired percentile of the Monte Carlo distribution of portfolio return.